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How to find displacement from velocity calculus

If you're seeing this message, it means we're having trouble loading external resources on our website. If you're behind a web filter, please make sure that the domains *.kastatic.org and *.kasandbox.org are unblocked. This section assumes you have enough background in calculus to be familiar with integration. In Instantaneous Velocity and Speed and Average and Instantaneous Acceleration we introduced the kinematic functions of velocity and acceleration using the derivative. By taking the derivative of the position function we found the velocity function, and likewise by taking the derivative of the velocity function we found the acceleration function. Using integral calculus, we can work backward and calculate the velocity function from the acceleration function, and the position function from the velocity function. Let's begin with a particle with an acceleration a(t) is a known function of time. Since the time derivative of the velocity function is acceleration, [latex] \frac{d}{dt}v(t)=a(t), [/latex] we can take the indefinite integral of both sides, finding [latex] \int \frac{d}{dt}v(t)dt=\int a(t)dt+{C}_{1}, [/latex] where C1 is a constant of integration. Since [latex] \int \frac{d}{dt}v(t)dt=v(t) [/latex], the velocity is given by [latex] v(t)=\int a(t)dt+{C}_{1}. [/latex] Similarly, the time derivative of the position function is the velocity function, [latex] \frac{d}{dt}x(t)=v(t). [/latex] Thus, we can use the same mathematical manipulations we just used and find [latex] x(t)=\int v(t)dt+{C}_{2}, [/latex] where C2 is a second constant of integration. We can derive the kinematic equations for a constant acceleration using these integrals. With a(t) = a a constant, and doing the integration in (Figure), we find [latex] v(t)=\int adt+{C}_{1}=at+{C}_{1}. [/latex] If the initial velocity is v(0) = v0, then [latex] {v}_{0}=0+{C}_{1}. [/latex] Then, C1 = v0 and [latex] v(t)={v}_{0}+at, [/latex] which is (Equation). Substituting this expression into (Figure) gives [latex] x(t)=\int ({v}_{0}+at)dt+{C}_{2}. [/latex] Doing the integration, we find [latex] x(t)={v}_{0}t+\frac{1}{2}a{t}^{2}+{C}_{2}. [/latex] If x(0) = x0, we have [latex] {x}_{0}=0+0+{C}_{2}; [/latex] so, C2 = x0. Substituting back into the equation for x(t), we finally have [latex] x(t)={x}_{0}+{v}_{0}t+\frac{1}{2}a{t}^{2}, [/latex] which is (Equation). A motorboat is traveling at a constant velocity of 5.0 m/s when it starts to decelerate to arrive at the dock. Its acceleration is [latex] a(t)=-\frac{1}{4}t\,\text{m/}{\text{s}}^{2} [/latex]. (a) What is the velocity function of the motorboat? (b) At what time does the velocity reach zero? (c) What is the position function of the motorboat? (d) What is the displacement of the motorboat from the time it begins to decelerate to when the velocity is zero? (e) Graph the velocity and position functions. Strategy (a) To get the velocity function we must integrate and use initial conditions to find the constant of integration. (b) We set the velocity function equal to zero and solve for t. (c) Similarly, we must integrate to find the position function and use initial conditions to find the constant of integration. (d) Since the initial position is taken to be zero, we only have to evaluate the position function at [latex] t=0 [/latex]. Solution We take t = 0 to be the time when the boat starts to decelerate. From the functional form of the acceleration we can solve (Figure) to get v(t): Solve (Figure): Figure 3.30 (a) Velocity of the motorboat as a function of time. The motorboat decreases its velocity to zero in 6.3 s. At times greater than this, velocity becomes negative—meaning, the boat is reversing direction. (b) Position of the motorboat as a function of time. At t = 6.3 s, the velocity is zero and the boat has stopped. At times greater than this, the velocity becomes negative—meaning, if the boat continues to move with the same acceleration, it reverses direction and heads back toward where it originated. Significance The acceleration function is linear in time so the integration involves simple polynomials. In (Figure), we see that if we extend the solution beyond the point when the velocity is zero, the velocity becomes negative and the boat reverses direction. This tells us that solutions can give us information outside our immediate interest and we should be careful when interpreting them. A particle starts from rest and has an acceleration function [latex] 5-10t{\text{m/s}}^{2} [/latex]. (a) What is the velocity function? (b) What is the position function? (c) When is the velocity zero? The acceleration of a particle varies with time according to the equation [latex] a(t)=p{t}^{2}-q{t}^{3} [/latex]. Initially, the velocity and position are zero. (a) What is the velocity as a function of time? (b) What is the position as a function of time? Between t = 0 and t = t0, a rocket moves straight upward with an acceleration given by [latex] a(t)=A-B{t}^{1\,\text{/}2} [/latex], where A and B are constants. (a) If x is in meters and t is in seconds, what are the units of A and B? (b) If the rocket starts from rest, how does the velocity vary between t = 0 and t = t0? (c) If its initial position is zero, what is the rocket's position as a function of time during this same time interval? The velocity of a particle moving along the x-axis varies with time according to [latex] v(t)=A+B{t}^{-1} [/latex], where A = 2 m/s, B = 0.25 m, and [latex] 1.0\,\text{s}\le t\le 8.0\,\text{s} [/latex]. Determine the acceleration and position of the particle at t = 2.0 s and t = 5.0 s. Assume that [latex] x(t=1\,\text{s})=0 [/latex]. A particle at rest leaves the origin with its velocity increasing with time according to v(t) = 3.2t m/s. At 5.0 s, the particle's velocity starts decreasing according to [16.0 – 1.5(t – 5.0)] m/s. This decrease continues until t = 11.0 s, after which the particle's velocity remains constant at 7.0 m/s. (a) What is the acceleration of the particle as a function of time? (b) What is the position of the particle at t = 2.0 s, t = 7.0 s, and t = 12.0 s? Professional baseball player Nolan Ryan could pitch a baseball at approximately 160.0 km/h. At that average velocity, how long did it take a ball thrown by Ryan to reach home plate, which is 18.4 m from the pitcher's mound? Compare this with the average reaction time of a human to a visual stimulus, which is 0.25 s. An airplane leaves Chicago and makes the 3000-km trip to Los Angeles in 5.0 h. A second plane leaves Chicago one-half hour later and arrives in Los Angeles at the same time. Compare the average velocities of the two planes. Ignore the curvature of Earth and the difference in altitude between the two cities. Unreasonable Results A cyclist rides 16.0 km east, then 8.0 km west, then 8.0 km east, then 32.0 km west, and finally 11.2 km east. If his average velocity is 24 km/h, how long did it take him to complete the trip? Is this a reasonable time? An object has an acceleration of [latex] +1.2\,{\text{cm/s}}^{2} [/latex]. At [latex] t=4.0\,\text{s} [/latex], its velocity is [latex] -3.4\,\text{cm/s} [/latex]. Determine the object's velocities at [latex] t=1.0\,\text{s} [/latex] and [latex] t=6.0\,\text{s} [/latex]. A particle moves along the x-axis according to the equation [latex] x(t)=2.0-4.0{t}^{2} [/latex] m. What are the velocity and acceleration at [latex] t=2.0 [/latex] s and [latex] t=5.0 [/latex] s? A particle moving at constant acceleration has velocities of [latex] 2.0\,\text{m/s} [/latex] at [latex] t=2.0 [/latex] s and [latex] -7.6\,\text{m/s} [/latex] at [latex] t=5.2 [/latex] s. What is the acceleration of the particle? A train is moving up a steep grade at constant velocity (see following figure) when its caboose breaks loose and starts rolling freely along the track. After 5.0 s, the caboose is 30 m behind the train. What is the acceleration of the caboose? An electron is moving in a straight line with a velocity of [latex] 4.0\,×\,{10}^{5} [/latex] m/s. It enters a region 5.0 cm long where it undergoes an acceleration of [latex] 6.0\,×\,{10}^{12}\,{\text{m/s}}^{2} [/latex] along the same straight line. (a) What is the electron's velocity when it emerges from this region? b) How long does the electron take to cross the region? An ambulance driver is rushing a patient to the hospital. While traveling at 72 km/h, she notices the traffic light at the upcoming intersections has turned amber. To reach the intersection before the light turns red, she must travel 50 m in 2.0 s. (a) What minimum acceleration must the ambulance have to reach the intersection before the light turns red? (b) What is the speed of the ambulance when it reaches the intersection? A motorcycle that is slowing down uniformly covers 2.0 successive km in 80 s and 120 s, respectively. Calculate (a) the acceleration of the motorcycle and (b) its velocity at the beginning and end of the 2-km trip. A cyclist travels from point A to point B in 10 min. During the first 2.0 min of her trip, she maintains a uniform acceleration of [latex] 0.090\,{\text{m/s}}^{2} [/latex]. She then travels at constant velocity for the next 5.0 min. Next, she decelerates at a constant rate so that she comes to a rest at point B 3.0 min later. (a) Sketch the velocity-versus-time graph for the trip. (b) What is the acceleration during the last 3 min? (c) How far does the cyclist travel? Two trains are moving at 30 m/s in opposite directions on the same track. The engineers see simultaneously that they are on a collision course and apply the brakes when they are 1000 m apart. Assuming both trains have the same acceleration, what must this acceleration be if the trains are to stop just short of colliding? A 10.0-m-long truck moving with a constant velocity of 97.0 km/h passes a 3.0-m-long car moving with a constant velocity of 80.0 km/h. How much time elapses between the moment the front of the truck is even with the back of the car and the moment the back of the truck is even with the front of the car? A police car waits in hiding slightly off the highway. A speeding car is spotted by the police car doing 40 m/s. At the instant the speeding car passes the police car, the police car accelerates from rest at 4 m/s2 to catch the speeding car. How long does it take the police car to catch the speeding car? Pablo is running in a half marathon at a velocity of 3 m/s. Another runner, Jacob, is 50 meters behind Pablo with the same velocity. Jacob begins to accelerate at 0.05 m/s2. (a) How long does it take Jacob to catch Pablo? (b) What is the distance covered by Jacob? (c) What is the final velocity of Jacob? Unreasonable results A runner approaches the finish line and is 75 m away; her average speed at this position is 8 m/s. She decelerates at this point at 0.5 m/s2. How long does it take her to cross the finish line from 75 m away? Is this reasonable? An airplane accelerates at 5.0 m/s2 for 30.0 s. During this time, it covers a distance of 10.0 km. What are the initial and final velocities of the airplane? Compare the distance traveled of an object that undergoes a change in velocity that is twice its initial velocity with an object that changes its velocity by four times its initial velocity over the same time period. The accelerations of both objects are constant. An object is moving east with a constant velocity and is at position [latex] {x}_{0}\,\text{at}\,\text{time}\,{t}_{0}=0 [/latex]. (a) With what acceleration must the object have for its total displacement to be zero at a later time t ? (b) What is the physical interpretation of the solution in the case for [latex] t\to \infty [/latex]? A ball is thrown straight up. It passes a 2.00-m-high window 7.50 m off the ground on its path up and takes 1.30 s to go past the window. What was the ball's initial velocity? A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground. A soft tennis ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.10 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 3.50 ms [latex] (3.50\,×\,{10}^{-3}\,\text{s}) [/latex] (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid? Unreasonable results. A raindrop falls from a cloud 100 m above the ground. Neglect air resistance. What is the speed of the raindrop when it hits the ground? Is this a reasonable number? Compare the time in the air of a basketball player who jumps 1.0 m vertically off the floor with that of a player who jumps 0.3 m vertically. Suppose that a person takes 0.5 s to react and move his hand to catch an object he has dropped. (a) How far does the object fall on Earth, where [latex] g=9.8\,{\text{m/s}}^{2}? [/latex] (b) How far does the object fall on the Moon, where the acceleration due to gravity is 1/6 of that on Earth? A hot-air balloon rises from ground level at a constant velocity of 3.0 m/s. One minute after liftoff, a sandbag is dropped accidentally from the balloon. Calculate (a) the time it takes for the sandbag to reach the ground and (b) the velocity of the sandbag when it hits the ground. (a) A world record was set for the men's 100-m dash in the 2008 Olympic Games in Beijing by Usain Bolt of Jamaica. Bolt "coasted" across the finish line with a time of 9.69 s. If we assume that Bolt accelerated for 3.00 s to reach his maximum speed, and maintained that speed for the rest of the race, calculate his maximum speed and his acceleration. (b) During the same Olympics, Bolt also set the world record in the 200-m dash with a time of 19.30 s. Using the same assumptions as for the 100-m dash, what was his maximum speed for this race? An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground. A steel ball is dropped onto a hard floor from a height of 1.50 m and rebounds to a height of 1.45 m. (a) Calculate its velocity just before it strikes the floor. (b) Calculate its velocity just after it leaves the floor on its way back up. (c) Calculate its acceleration during contact with the floor if that contact lasts 0.0800 ms [latex] (8.00\,×\,{10}^{-5}\,\text{s}) [/latex] (d) How much did the ball compress during its collision with the floor, assuming the floor is absolutely rigid? An object is dropped from a roof of a building of height h. During the last second of its descent, it drops a distance h/3. Calculate the height of the building. Page 2 Up until this point we have looked at examples of motion involving a single body. Even for the problem with two cars and the stopping distances on wet and dry roads, we divided this problem into two separate problems to find the answers. In a two-body pursuit problem, the motions of the objects are coupled—meaning, the unknown we seek depends on the motion of both objects. To solve these problems we write the equations of motion for each object and then solve them simultaneously to find the unknown. This is illustrated in (Figure). Figure 3.25 A two-body pursuit scenario where car 2 has a constant velocity and car 1 is behind with a constant acceleration. Car 1 catches up with car 2 at a later time. The time and distance required for car 1 to catch car 2 depends on the initial distance car 1 is from car 2 as well as the velocities of both cars and the acceleration of car 1. The kinematic equations describing the motion of both cars must be solved to find these unknowns. Consider the following example. A cheetah waits in hiding behind a bush. The cheetah spots a gazelle running past at 10 m/s. At the instant the gazelle passes the cheetah, the cheetah accelerates from rest at 4 m/s2 to catch the gazelle. (a) How long does it take the cheetah to catch the gazelle? (b) What is the displacement of the gazelle and cheetah? Strategy We use the set of equations for constant acceleration to solve this problem. Since there are two objects in motion, we have separate equations of motion describing each animal. But what links the equations is a common parameter that has the same value for each animal. If we look at the problem closely, it is clear the common parameter to each animal is their position x at a later time t. Since they both start at [latex] {x}_{0}=0 [/latex], their displacements are the same at a later time t, when the cheetah catches up with the gazelle. If we pick the equation of motion that solves for the displacement for each animal, we can then set the equations equal to each other and solve for the unknown, which is time. Solution Significance It is important to analyze the motion of each object and to use the appropriate kinematic equations to describe the individual motion. It is also important to have a good visual perspective of the two-body pursuit problem to see the common parameter that links the motion of both objects. A bicycle has a constant velocity of 10 m/s. A person starts from rest and runs to catch up to the bicycle in 30 s. What is the acceleration of the person? A particle moves in a straight line at a constant velocity of 30 m/s. What is its displacement between t = 0 and t = 5.0 s? A particle moves in a straight line with an initial velocity of 30 m/s and a constant acceleration of 30 m/s2. If at [latex] t=0,x=0 [/latex] and [latex] v=0 [/latex], what is the particle's position at t = 5 s? A particle moves in a straight line with an initial velocity of 30 m/s and constant acceleration 30 m/s2. (a) What is its displacement at t = 5 s? (b) What is its velocity at this same time? (a) Sketch a graph of velocity versus time corresponding to the graph of displacement versus time given in the following figure. (b) Identify the time or times (ta, tb, tc, etc.) at which the instantaneous velocity has the greatest positive value. (c) At which times is it zero? (d) At which times is it negative? (a) Sketch a graph of acceleration versus time corresponding to the graph of velocity versus time given in the following figure. (b) Identify the time or times (ta, tb, tc, etc.) at which the acceleration has the greatest positive value. (c) At which times is it zero? (d) At which times is it negative? A particle has a constant acceleration of 6.0 m/s2. (a) If its initial velocity is 2.0 m/s, at what time is its displacement 5.0 m? (b) What is its velocity at that time? At t = 10 s, a particle is moving from left to right with a speed of 5.0 m/s. At t = 20 s, the particle is moving right to left with a speed of 8.0 m/s. Assuming the particle's acceleration is constant, determine (a) its acceleration, (b) its initial velocity, and (c) the instant when its velocity is zero. A well-thrown ball is caught in a well-padded mitt. If the acceleration of the ball is[latex] 2.10\,×\,{10}^{4}{\,\text{m/s}}^{2} [/latex], and 1.85 ms [latex] (1\,\text{ms}={10}^{-3}\,\text{s}) [/latex] elapses from the time the ball first touches the mitt until it stops, what is the initial velocity of the ball? A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of [latex] 6.20\,×\,{10}^{5}{\,\text{m/s}}^{2} [/latex] for [latex] 8.10\,×\,{10}^{\text{−}4}\,\text{s} [/latex]. What is its muzzle velocity (that is, its final velocity)? (a) A light-rail commuter train accelerates at a rate of 1.35 m/s2. How long does it take to reach its top speed of 80.0 km/h, starting from rest? (b) The same train ordinarily decelerates at a rate of 1.65 m/s2. How long does it take to come to a stop from its top speed? (c) In emergencies, the train can decelerate more rapidly, coming to rest from 80.0 km/h in 8.30 s. What is its emergency acceleration in meters per second squared? While entering a freeway, a car accelerates from rest at a rate of 2.04 m/s2 for 12.0 s. (a) Draw a sketch of the situation. (b) List the knowns in this problem. (c) How far does the car travel in those 12.0 s? To solve this part, first identify the unknown, then indicate how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, check your units, and discuss whether the answer is reasonable. (d) What is the car's final velocity? Solve for this unknown in the same manner as in (c), showing all steps explicitly. Unreasonable results At the end of a race, a runner decelerates from a velocity of 9.00 m/s at a rate of 2.00 m/s2. (a) How far does she travel in the next 5.00 s? (b) What is her final velocity? (c) Evaluate the result. Does it make sense? Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat? During a slap shot, a hockey player accelerates the puck from a velocity of 8.00 m/s to 40.0 m/s in the same direction. If this shot takes [latex] 3.33\,×\,{10}^{\text{−}2}\,\text{s} [/latex], what is the distance over which the puck accelerates? A powerful motorcycle can accelerate from rest to 26.8 m/s (100 km/h) in only 3.90 s. (a) What is its average acceleration? (b) How far does it travel in that time? Freight trains can produce only relatively small accelerations. (a) What is the final velocity of a freight train that accelerates at a rate of [latex] 0.0500\,{\text{m/s}}^{2} [/latex] for 8.00 min, starting with an initial velocity of 4.00 m/s? (b) If the train can slow down at a rate of [latex] 0.550\,{\text{m/s}}^{2} [/latex], how long will it take to come to a stop from this velocity? (c) How far will it travel in each case? A fireworks shell is accelerated from rest to a velocity of 65.0 m/s over a distance of 0.250 m. (a) Calculate the acceleration. (b) How long did the acceleration last? A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of [latex] 0.35\,{\text{m/s}}^{2} [/latex], how far will it travel before becoming airborne? (b) How long does this take? A woodpecker's brain is specially protected from large accelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in meters per second squared and in multiples of g, where g = 9.80 m/s2. (b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less acceleration of the brain). What is the brain's acceleration, expressed in multiples of g? An unwary football player collides with a padded goalpost while running at a velocity of 7.50 m/s and comes to a full stop after compressing the padding and his body 0.350 m. (a) What is his acceleration? (b) How long does the collision last? A care package is dropped out of a cargo plane and lands in the forest. If we assume the care package speed on impact is 54 m/s (123 mph), then what is its acceleration? Assume the trees and snow stops it over a distance of 3.0 m. An express train passes through a station. It enters with an initial velocity of 22.0 m/s and decelerates at a rate of [latex] 0.150\,{\text{m/s}}^{2} [/latex] as it goes through. The station is 210.0 m long. (a) How fast is it going when the nose leaves the station? (b) How long is the nose of the train in the station? (c) If the train is 130 m long, what is the velocity of the end of the train as it leaves? (d) When does the end of the train leave the station? Unreasonable results Dragsters can actually reach a top speed of 145.0 m/s in only 4.45 s. (a) Calculate the average acceleration for such a dragster. (b) Find the final velocity of this dragster starting from rest and accelerating at the rate found in (a) for 402.0 m (a quarter mile) without using any information on time. (c) Why is the final velocity greater than that used to find the average acceleration? (Hint: Consider whether the assumption of constant acceleration is valid for a dragster. If not, discuss whether the acceleration would be greater at the beginning or end of the run and what effect that would have on the final velocity.)